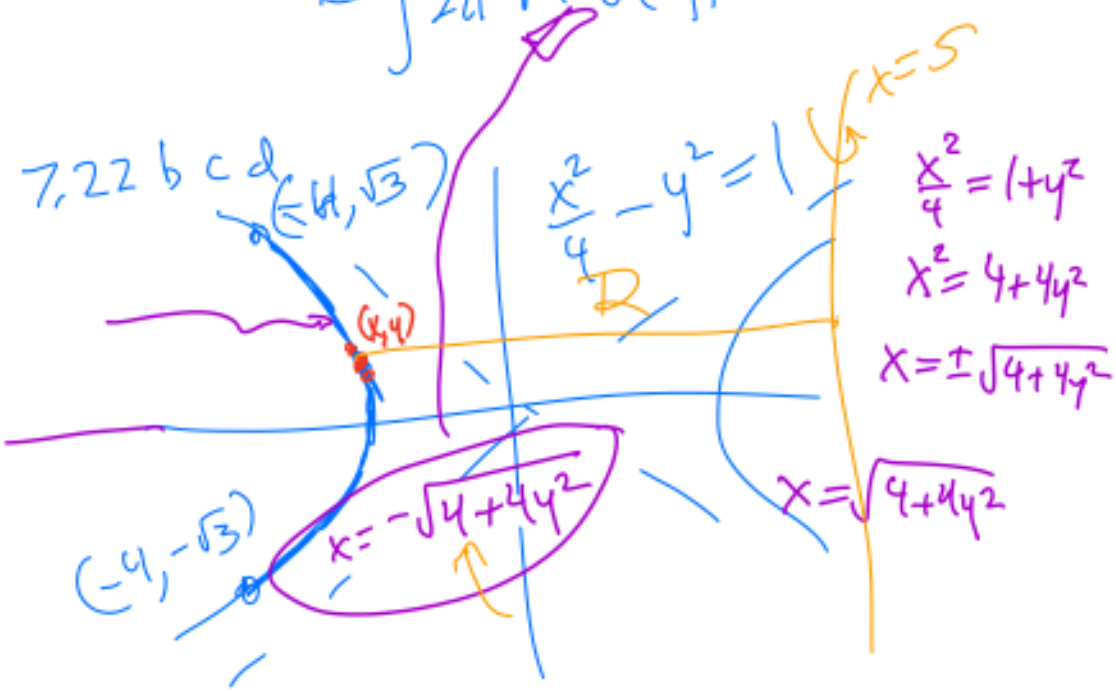
$$\text{Surface area} = \int 2\pi R\, l = \int 2\pi R \sqrt{dx^2 + dy^2}$$

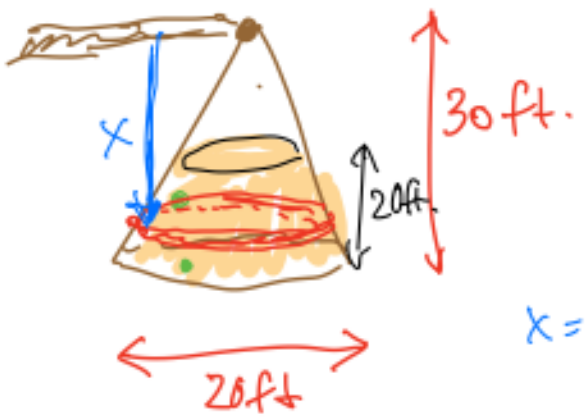
$$= \int 2\pi R \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx.$$

$$= \int 2\pi R \sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\, dy$$

7.22 b c d



Example A tank in the shape of a cone has a diameter of 20 ft and a height of 30 ft. It is filled with gasoline to a level of 20 ft. How much work is required to pump all of the gasoline to the top of the cone.



Gas at each level goes a different distance.

x = distance from top to slice

thickness of slice = Δx.

Force (weight) of slice = (Density of gas) · (Volume of slice)

→ 47 lbs/ft^3 ↑ ft^3

$\pi r^2 \Delta x$

slice 

side view 

small triangles

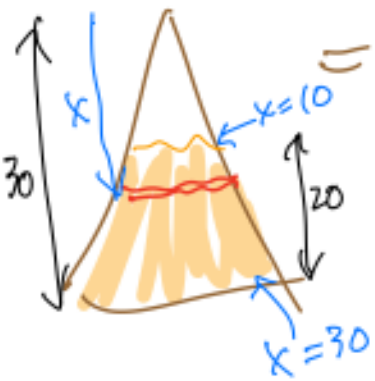
$$\frac{r}{x} = \frac{10}{30} = \frac{1}{3}$$

similar triangle $r = \frac{1}{3}x$

$$(\text{Force of slice}) = \left(47 \frac{\text{lbs}}{\text{ft}^3}\right) \pi \left(\frac{1}{3}x\right)^2 \Delta x = \frac{47}{9} \pi x^2 \Delta x$$

$$\left(\begin{matrix}\text{Vertical}\\ \text{Distance slice travels}\end{matrix}\right) = x$$

$$\text{Work} = \int (\text{Force})(\text{Distance})$$



$$= \int_{x=10}^{30} \left(\frac{47}{9}\pi x^2\, dx\right) x = \frac{47\pi}{9}\int_{10}^{30} x^3\, dx$$

$$= \frac{47\pi}{9}\left.\frac{x^4}{4}\right|_{10}^{30}$$

$$= \frac{47\pi}{9}\left(\frac{810000}{4} - \frac{10000}{4}\right)$$

$$= \frac{47\pi}{9}\left(\frac{800{,}000}{4}\right) =$$

$$\frac{47\pi}{9}(200000) \text{ ft lbs}$$

total work required.

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Quiz

(1) What is your name?

(2) What is your favorite ice cream?

(3) Simplify $\dfrac{x^5(x^3+x^2)}{5x^7}$

(4) Take the derivative:

(a) $\sin(5x^2+2)$

(b) $\dfrac{e^x}{x}$